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# Throughput Estimation of Manufacturing Systems with Mixed Blocking Types

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**Abstract.** This study proposed a decomposed method to calculate the throughput of manufacturing systems with mixed blocking types, including blocking before service and blocking after service. In particular, a manufacturing system is decomposed into sub-models, which decreases the analysis complexity of the steady-state probability of the manufacturing system. The iterative calculation formulations for blocking before service and blocking after service are proposed to deal with mixed blocking types and approximate the steady-state probability of the manufacturing system by iteratively calculating the state probabilities of the sub-models. If the state probability of the sub-model with output converges after several iterations, the throughput of the manufacturing system can be obtained based on the steady-state probability of the sub-model. To verify the effectiveness of the proposed decomposition method, it was compared to a simulator in a numerical example. The results demonstrated the effectiveness of the proposed method.

**Keywords:** throughput estimation, manufacturing system, mixed blocking type, iterative calculation formulation

## 1. Introduction

Currently, manufacturing systems have been rapidly building complex operation processes to achieve more powerful applications. Along with this tendency, it is becoming increasingly difficult to model manufacturing systems, analyze uncertainties and inter-dependencies in the systems, and predict system behavior. Therefore, performance evaluation, including throughput estimation, has been studied for the design, optimization, and management of manufacturing systems. A manufacturing system consists of materials, machines, and buffers. Because the buffers provide finite storage space, blocking may occur where the flow of materials through them, leading to a momentarily stop. Blocking not only decreases system working efficiency but also causes deadlocks in a manufacturing system [1]. To describe the blocking of the materials, blocking types are defined. A manufacturing system typically contains one blocking type. However, for the manufacturing system with a complex operation process, mixed blocking types may exist and increase the analysis difficulty of the system significantly. Therefore, it is necessary to develop a throughput estimation method for a manufacturing system with mixed blocking types.

While there exist several blocking types, blocking before service (BBS) and before after service (BAS) types are commonly used in manufacturing systems [2]. BAS is defined as the jobs upon completion of service in machine 1 cannot enter buffer 2 in the case of no available space in buffer 2. If the space in buffer 2 becomes available, the jobs in machine 1 enter buffer 2 immediately. Previous studies have proposed many analytical methods to solve the throughput estimation problem of manufacturing systems with the BAS. The exact solution for manufacturing systems was proposed firstly based on two stations model [2-5]; however, its long computation time limited the application for large-scale manufacturing systems. To decrease the computation complexity and reduce computation time, decomposition methods [6-9] were proposed to evaluate the throughput of manufacturing systems. Different from the BAS, the BBS is defined as the jobs in machine 1 cannot receive service in the case of no available space in buffer 2. If the space in buffer 2 becomes available, the jobs in machine 1 start to receive service and enter buffer 2 upon completion of the service. The exact

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solutions for BBS have been proposed [2]; however, decomposition methods are more applicable to manufacturing systems with complex operation processes under the BBS. Brandwajn and Jow [10] firstly proposed a decomposition method for the tandem system considering the blocking mechanism of BBS. Schmidt and Jackman [11] improved the decomposition method and applied it to systems with close loops. Gao et al. [12] proposed modular queues based on the decomposition method to decrease computation time and extend its application for manufacturing systems with different configurations.

The above studies mainly focused on the throughput estimation of the manufacturing systems with one blocking type, which does not apply to manufacturing systems with mixed blocking types. To solve this problem, simulation methods [13-15] were proposed. However, compared to analytical methods, simulation methods usually require longer modeling and simulation time [16]. In addition, the throughput estimation is not only used for the performance evaluation of manufacturing systems but also used for the optimization of the manufacturing systems, such as the buffer allocation problem. In the optimization of the manufacturing systems, the throughput estimation is utilized a large number of times, limiting the application of the simulation methods further [17]. Therefore, the objective of this study is to propose an analytic throughput estimation method for manufacturing systems with mixed blocking types, including the BBS and BAS.

In this study, a decomposition method is proposed to calculate the throughput of manufacturing systems with mixed BBS and BAS. A manufacturing system is firstly decomposed into sub-models, including three stations, which changes the analysis of the complex steady-state probability of the manufacturing system to the analysis of the steady-state probabilities of the sub-models. The iterative calculation formulations (ICFs) for the mixed BBS and BAS are proposed to iteratively calculate the state probabilities of the sub-models until the state probability of the sub-model with output converges. The throughput of the manufacturing system is obtained based on the state probability of the sub-model with output.

The remainder of this paper is structured as follows. Section 2 presents the problem statement. Section 3 presents the methodology. Numerical examples are illustrated to verify the effectiveness of the proposed methodology in Section 4. Section 5 provides some conclusions and directions for future work.

## 2. Problem Statement

### 2.1. Model and Assumptions

We model a manufacturing system as the stochastic model that consists of a number of stations, as shown in Fig. 1. Each station has one machine and one buffer. All the assumptions are as follows:

- Each machine has its handling capability denoted by the service rate with an exponential distribution.
- Each buffer has a finite capacity.
- Materials are discrete jobs.
- Jobs enter a manufacturing system at an external arrival rate according to Poisson's process. In addition, the behaviors of the external arrival jobs are described as they only enter the manufacturing system with the external arrival rate in the case of the available space in the first station, while the external jobs stop arriving in the case of no available space.
- In a manufacturing system, the first machine $M_1$ is never starved; meanwhile, the final machine $M_I$ is never blocked.

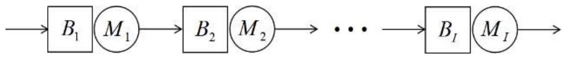


Fig. 1: Manufacturing system involving $I$ stations

### 2.2. Parameters

The input parameters are as follows:

- $\lambda$: denotes the external arrival rate at which jobs enter a manufacturing system per unit time.
- $a_i$: denotes the effective arrival rate at which jobs enter station $i$ per unit time because of the blocking between station $i$-$1$ and station $i$.
- $\mu_i$: denotes the service rate at which jobs leave station $i$ per unit time.
- $u_i$: denotes the effective service rate at which jobs leave station $i$ because of the blocking between station $i$ and station $i$+$1$.

- $i$: denotes a station in a manufacturing system. $I$ is the total number of stations.
- $K_i$: denotes the capability of the buffer in station $i$.

The output parameter is $T$ which denotes the throughput rate at which jobs exit a manufacturing system per unit time.

# 3. Methodology

The study proposes a decomposition method to calculate the throughput of a manufacturing system with mixed blocking types, including the BBS and BAS. Fig. 2 shows the framework of the proposed method. The manufacturing system is firstly decomposed into sub-models. Previous studies typically analyzed the state probabilities of the sub-model with only one blocking type. In this study, the ICFs for the mixed BBS and BAS are proposed to iteratively calculate state probabilities of the sub-models with mixed blocking types until the steady-state probability of the sub-model with output is obtained. Finally, the throughput of the manufacturing system is obtained based on the steady-state probability of the sub-model with output.

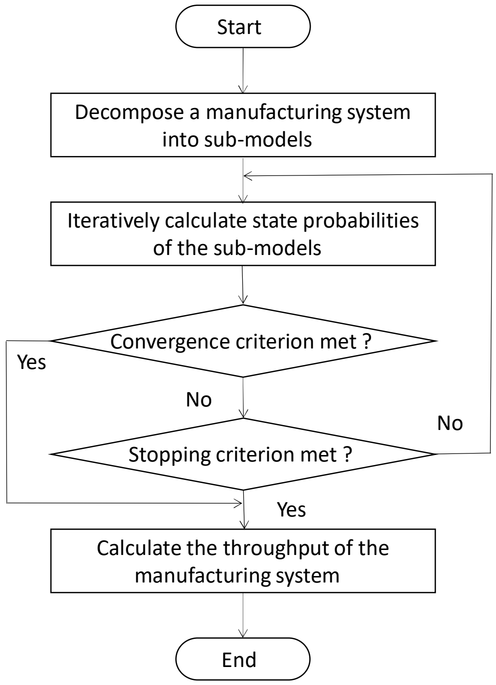


Fig. 2: Framework of the proposed method.

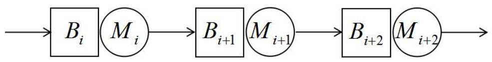


Fig. 3: Sub-model with the BBS.

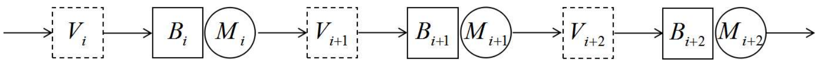


Fig. 4: Sub-model with the BAS.

## 3.1. The ICFs for the mixed BBS and BAS

Fig. 3 shows a sub-model with the BBS. Based on the BBS mechanism, jobs in station $i$ stop receiving service if there is no available space in station $i+1$. After the space in station $i+1$ is available, the jobs in station $i$ start receiving service and move to station $i+1$ upon completion of service. The previous study [10] proposed an iterative calculation formulation for a queuing system with the BBS. We apply the formulation to sub-model $i$-$(i+1)$-$(i+2)$ to obtain the formulation for the BBS in the literature [12]. In addition, in a blocked station with the BBS, the delay for blocked jobs to enter its following station is fixed to the service time (reciprocal of service rate) of the blocked station if the space in its following station is available. However, jobs in a blocked station with the BAS continue receiving service, even though no available space in its following station. When the space is available in its next station, the blocked jobs may be in service or already serviced. Therefore, the delay for the blocked jobs to enter their following station is variable. Considering the BAS mechanism, we propose the station model with the BAS by improving the former station model with the BBS. A visual buffer is set before each station to approximate the variable delay of the BAS. Fig. 4 shows a sub-model with the BAS. The blocked jobs behavior can be described as the jobs in station $i$ continue receiving service even though there is no available space in station $i+1$. Jobs upon completion of service move to visual buffer $i+1$. After the space in station $i+1$ is available, jobs in visual buffer $i+1$move to the station immediately. The capacity of the visual buffer is based on the number of machines in station $i$. In the study, it is supposed that each station only contains one machine. Therefore, the capacity of the visual buffer is set to "1". Based on the former formulations for the BBS, we merge the buffer in a station and the visual buffer before it. Considering the formulation for the BBS, the ICFs for the mixed BBS and BAS are defined as follows:

$$d_i^j(n_i,n_{i+1})=\begin{cases}\lambda, i=1\\ \sum_{n_{i-1}=1}^{k_{i-1}}\mu_{i-1}\,\pi(n_{i-1}|n_i,n_{i+1}), i=1\cdots I-2\end{cases} \tag{1}$$

$$u_{i+2}^j(n_{i+1},n_{i+2})=\begin{cases}\mu_{i+2}, i=I-2\\ \sum_{n_{i+3}=0}^{k_{i+3}-1}\mu_{i+2}\,\pi(n_{i+3}|n_{i+1},n_{i+2}), i=1\cdots I-3\end{cases} \tag{2}$$

$$k_i=\begin{cases}K_i, BBS\\ K_i+1, BAS\end{cases} \tag{3}$$

where $j$ denotes the number of iterations; $\pi^j(n_{i-1}|n_i,n_{i+1})$ denotes the conditional probability of $n_{i-1}$ jobs being in station $i-1$ given that $n_i$ and $n_{i+1}$ jobs being in station $i$ and $i+1$, respectively, in iteration $j$; $k_i$ is the parameter to distinguish blocking type in station $i$.

After $d_i^j$ and $u_{i+2}^j$ are obtained, $\pi^j(n_i,n_{i+1},n_{i+2})$ is obtained by substituting $d_i^j$, $u_{i+2}^j$, $\mu_i$, and $\mu_{i+1}$ into global balance equations (GBEs) and normalization equations (NEs) [1]. $\pi^j(n_i,n_{i+1},n_{i+2})$ can be used to calculate $d_{i+1}^{j+1}$ for the sub-model $(i+1)$-$(i+2)$-$(i+3)$ and $u_{i+1}^{j+1}$ for the sub-model $(i-1)$-$i$-$(i+1)$ in iteration $j+1$ using (1), (2), and (3). The iteration calculation process keeps until $\pi^c(n_{I-2},n_{I-1},n_I)$ for the sub-model with output $(I-2)$-$(I-1)$-$I$ converges in iteration $c$ based on the convergence criterion.

A commonly used convergence criterion is used [11,18], even though its proof is not discussed in detail. The convergence criterion is defined as follows:

$$\left|\frac{\pi^c(n_{I-2},n_{I-1},n_I)-\pi^{c-1}(n_{I-2},n_{I-1},n_I)}{\pi^{c-1}(n_{I-2},n_{I-1},n_I)}\right|\le 0.0001 \tag{4}$$

In addition, to avoid an infinite iteration calculation loop, a stopping criterion is set. Based on our previous experiments, the allowed maximum number of iterations is 100.

### 3.2. Iterative calculation process

The specific iterative calculation process is as follows:

- Step 1: Initialize input parameters $\lambda$, $\mu_i$, $K_i$, $I$, and state probabilities $\pi^0(n_i,n_{i+1},n_{i+2}), i=1\cdots I-2$. Set $j=1$.
- Step 2: Calculate $d_i^j$ and $u_{i+2}^j$ using (1), (2), and (3) for the sub-model $i$-$(i+1)$-$(i+2), i=1\cdots I-2$.
- Step 3: Substitute $d_i^j$, $\mu_i$, $\mu_{i+1}$, and $u_{i+2}^j$ into GBEs and NEs to obtain $\pi^j(n_i,n_{i+1},n_{i+2})$ for sub-model $i$-$(i+1)$-$(i+2)$.
- Step 4: Judge whether $\pi^j(n_{I-2},n_{I-1},n_I)$ have been obtained. If so, go to step 5; otherwise, return to step 2.
- Step 5: Judge whether $\pi^j(n_{I-2},n_{I-1},n_I)$ has converged. If so, stop; otherwise, go to step 6.
- Step 6: Judge whether the stopping criterion has been met. If so, stop; otherwise, set $j=j+1$ and return step 2.

### 3.3. Throughput calculation

If the state probability $\pi^c(n_{I-2},n_{I-1},n_I)$ converges in iteration $c$, the throughput of the manufacturing system can be obtained as follow:

$$T=\mu_I\sum_{n_I=1}^{K_I}\pi^c(n_{I-2},n_{I-1},n_I) \tag{5}$$

where $\mu_I$ denotes the service rate of machine $I$, and $\sum_{n_I=1}^{K_I}\pi^c(n_{I-2},n_{I-1},n_I)$ denotes the steady-state probability of jobs being in station $I$.

# 4. Numerical Example

This section presents numerical examples to verify the effectiveness of the proposed decomposition method. Input parameters consisted of external arrival rates, service rates of machines, the number of stations, and blocking type of stations. The capacity of buffers in the numerical examples was set to “1”. In addition, a discrete simulator based on Simpy was used as a benchmark to evaluate the accuracy of the proposed method in the numerical example. Relative errors were used to show the results differences between the proposed method and the simulator. It is defined as follows:

$$\delta=\frac{T_{analysis}-T_{simulator}}{T_{simulator}} \tag{6}$$

where $T_{analysis}$ denotes the throughput obtained by the proposed method, and $T_{simulator}$ denotes the data obtained by the simulator.

The proposed algorithm was written in Python 3.6.0 and executed for all experiments on a computer with a 2.3 GHz Intel Core 2 Duo CPU.

## 4.1. Setting

A manufacturing system involving five stations is tested. Table 1 lists the input parameters for the manufacturing system involving five stations. The manufacturing system with the mixed BBS and BAS is shown in Fig. 5.

Table 1: Input parameters for the manufacturing system involving five stations

| **Items** | **Parameters** |
|---|---|
| External arrival rate | $\lambda$ (jobs/s) |
| Service rates of stations | $\mu_1=\mu_2=\mu_3=\mu_4=\mu_a$ (jobs/s)<br>$\mu_5=\mu_b$ (jobs/s) |
| Number of stations | $I$=5 |
| Blocking types | Station 1, 2, 3, and 4 follow the BAS.<br>Station 5 follows the BBS. |

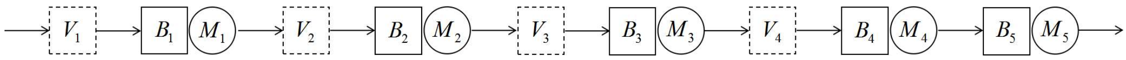


Fig. 5: Manufacturing system involving five stations with mixed BBS and BAS.

Table 2: Result comparison.

| **Input patterns ($\lambda$,$\mu_a$,$\mu_b$)** | **Proposed method $T_{analysis}$ (jobs/s)** | **Simulator $T_{simulator}$ (jobs/s)** | **Relative Errors $\delta$ (%)** |
|---|---|---|---|
| (1.00, 1.00, 0.50) | 0.3238 | 0.3332 | 2.82 |
| (0.75, 1.00, 0.50) | 0.3235 | 0.3329 | 2.82 |
| (0.50, 1.00, 0.50) | 0.3211 | 0.3303 | 2.78 |
| (0.25, 1.00, 0.50) | 0.2367 | 0.2423 | 2.31 |
| (0.10, 1.00, 0.50) | 0.0990 | 0.1000 | 1.00 |

## 4.2. Results

Table 2 lists the relative errors between the obtained throughput calculated by the proposed method and the data by the simulator. Fig. 6 shows the relationship between the relative errors and external arrival rates. The relative errors were lower than 5%. The relative errors became higher when the external arrival rate increased. The accuracy loss of the throughput estimation in the proposed method was mainly caused by decomposition. In the decomposition, not only the state probability of the manufacturing system but also job blocking behavior was decomposed. (1), (2), and (3) are proposed to iteratively calculate state probabilities of sub-models to approximate the state probability of the manufacturing system and transfer job blocking behaviors among the sub-models. In the case of a high external arrival rate, there were a large number of external jobs entering a system, and heavy blocking occurs in the manufacturing system. This resulted in more job blocking behaviors that were transferred among the sub-models, leading to higher relative errors. On the contrary, in the cases of a low external arrival rate, fewer blocking behaviors were transferred among the sub-models, and the differences between the proposed method and the simulator become smaller.

All the throughput obtained by the proposed method is lower than the data obtained by the simulator because the proposed method overestimates the blocking in the system and generates a lowly steady-state probability of jobs being an output station.

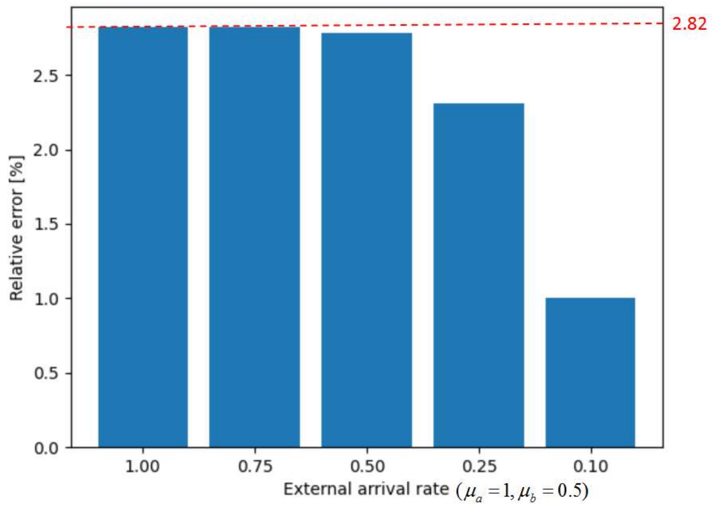


Fig. 6: Relationship between the relative errors and the external arrival rates.

### 4.3. Discussion

The proposed decomposition method calculates the throughput of a manufacturing system with the mixed BBS and BAS with an accuracy of ≥90%, although the accuracy decreases as the external arrival rate and the number of stations in the system increase. The accuracy decreases to 90% in the case of a high external arrival rate that causes more job blocking behaviors to be transferred among sub-models. However, the accuracy becomes high in the case of a low external arrival rate in which less blocking occurs.

Both the proposed method and the simulator can solve the throughput estimation of manufacturing systems with mixed blocking types. The simulator usually requires a long computation time for its environmental setting and simulation modeling, although it usually generates results that are close to real systems. This shortcoming may be enlarged if the simulator is used in the throughput estimation for the optimization of a manufacturing system. It is difficult for the simulator to achieve the balance between accuracy and computation time. However, the proposed method is more efficient in calculating the throughput with acceptable accuracy because of rapid modeling and no environmental setting, especially in the problem of the buffer allocation, server allocation, and service rate allocation of manufacturing systems.

## 5. Conclusions and future work

This study presented a decomposition method to solve the throughput estimation problem of manufacturing systems with mixed BBS and BAS. In the proposed method, sub-models were used to decompose a manufacturing system. The ICFs for mixed BBS and BAS were proposed to approximate the steady-state probability of the manufacturing system by iteratively calculating the state probabilities of the sub-models with mixed blocking types. To verify the effectiveness of the proposed method, it was compared to a simulator. The numerical examples show that the proposed method can achieve throughput estimation with an accuracy of ≥90% for manufacturing systems with mixed BBS and BAS.

The main implication of the study is to support the performance estimation and optimization problems of manufacturing systems with mixed BBS and BAS.

Because the study is in the first stage, some parameters in the proposed decomposition method have not been discussed in detail, such as variable buffer capacity and the variable number of machines in a station. In addition, the factors such as repair rate and failure rate are not discussed in the study. This may cause a gap between the study and the real manufacturing system. Therefore, another future work is to improve the proposed method to apply to unreliable manufacturing systems.

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